JUNE 13, 2003   Math 226 Midterm # 2

Fully answer all the questions in the booklets provided. Cl[...]
No communication of any kind with your fellow students. [...]
single $8\frac{1}{2} \times 11$ hand written sheet. Time: 2 hours.

Question 1. Determine the radii of convergence of each of the following power series.   10

(a) $\displaystyle\sum_{n=1}^{\infty} x^n \sin\left(\frac{1}{2^n}\right)$ (b) $\displaystyle\sum_{n=1}^{\infty} n!(x-5)^n$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{(n!)^3}$ (d) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{3^n\sqrt{n}}$

(e) $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n}}{4^n n^2}$

Question 2. Determine the interval of convergence of each of the following power series.   9

(a) $\displaystyle\sum_{n=1}^{\infty} \sqrt{n}x^n$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{x^{2n}}{n^2}$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{2^n(n+3)}$

Question 3. Suppose $f(x)$ is a function which has Taylor series   6

$$f(x) = 2x - \frac{1}{2}x^2 + 5x^3 + \cdots .$$

Calculate, up to and including terms of order x^3, the Taylor expansions of the functions

(a) $f(x)\sin x$ (b) $f(x) + 2\sqrt{1-x}$ (c) $xf'(x)$.

Question 4. Find the Taylor series, up to and including terms of order x^2, of the function   5

$$y = \ln(1 + 2\sin x).$$

Question 5. Find the general solution to the following differential equations.   6

(a) $\dfrac{dy}{dx} = \dfrac{y(x^2-4)}{x}$ (b) $\dfrac{1}{t}\dfrac{dy}{dt} - y + 2 = 0.$

Question 6. Find the general solution to the following differential equations.   6

(a) $\dfrac{dy}{dx} + \dfrac{4x}{1+x^2}y = \dfrac{4x}{1+x^2}$ (b) $x\dfrac{dy}{dx} + (1+x)y - 1 = 0.$

Question 7. Solve the following initial value problems.   6

(a) $\dfrac{dy}{dx} + y = x^2, \quad y(1) = 2$ (b) $\dfrac{dy}{dt} = \dfrac{y-1}{t-1}, \quad y(3) = 2.$